AP Calculus BC — Average Value of a Function

1) $\frac{1}{4}\int_2^6 f(x)\,dx = -1.672$

2) $\frac{1}{3}\int_{-1}^{2} f(x)\,dx = -4$ $\quad \frac{1}{5}\int_2^7 f(x)\,dx = 8$

$\int_{-1}^{2} f(x)\,dx = -12$ $\quad \int_2^7 f(x)\,dx = 40$

$\int_{-1}^{7} f(x)\,dx = 28$

$\frac{1}{8}\int f(x)\,dx = \frac{7}{2}$

3) $\frac{1}{2}\int_1^3 f(x)\,dx = 0.369$

4) $\frac{1}{2}\int_2^4 \frac{k}{x^2}\,dx$

$\frac{1}{2}\left[-kx^{-1}\right]_2^4$

$\frac{1}{2}\left[-\frac{k}{4} + \frac{k}{2}\right] = \boxed{\frac{k}{8}}$

5) a) $R'\left(\frac{3}{4}\right) \approx \frac{R(1)-R(\frac{1}{2})}{1-\frac{1}{2}}$

$= \frac{7-12}{\frac{1}{2}}$

$= -10\ \text{gal}/\text{hr}^2$

b) $\int_0^2 R(t)\,dt \approx$

$\frac{1}{4}\left[R(0) + 2R(\frac{1}{2}) + 2R(1) + 2R(\frac{3}{2}) + R(2)\right]$

$\frac{1}{4}\left[16 + 24 + 14 + 6 + 0\right]$ gallons

c) $\int_0^1 R'(t)\,dt$

$R(t)\Big|_0^1$

$R(1) - R(0)$

d) $\int_{1/2}^{3/2} R(t)\,dt$

6) a) $\int_0^9 f'(x)\,dx = 12 - 10 + 5 - 1$

$= 6$

b) $f(x) = 7 + \int_6^x f'(t)\,dt$

$x=0$	$x=6$	$x=9$
$f(0)=5$ MIN	$f(6)=7$	$f(9)=11$

c) $\int g(x)\,dx$

$\int (x^2-1)\,dx$

$\boxed{\frac{1}{3}x^3 - x + C}$

d) $\int_2^3 x f'(g(x))\,dx$

check: $2x f'(x^2-1)$

$f(g(x))$

$\left[\frac{1}{2} f(x^2-1)\right]_2^3$

$\frac{1}{2}f(8) - \frac{1}{2}f(3) = \boxed{\frac{1}{2}[2] - \frac{1}{2}[17]}$

7) $f(x) = 4x^{-2}$

a) $L_3 = f(1)\cdot 3 + f(4)\cdot 4 + f(8)\cdot 4$

$= (4)(3) + (\frac{1}{4})\cdot 4 + (\frac{1}{16})(4)$

b) $\int_1^{12} f(x)dx$

$\left[-4x^{-1}\right]\Big|_1^{12}$

$\boxed{-\frac{1}{3} + 4}$

c) $\int_1^{\infty} f(x)dx$

$\lim_{b\to\infty} \int_1^b 4x^{-2}dx$

$\lim_{b\to\infty} \left[-\frac{4}{x}\right]_1^b$

$\lim_{b\to\infty} \left[-\frac{4}{b} + 4\right] = \boxed{4}$

d) $\int_1^{12} 4x^{-2}\ln x\, dx$

$u = \ln x \qquad v = -4x^{-1}$

$du = \frac{1}{x} \qquad dv = 4x^{-2}dx$

$-\frac{4}{x}\ln x\Big|_1^{12} + 4\int_1^{12} \frac{1}{x^2}dx$

$-\frac{4}{x}\ln x\Big|_1^{12} + 4\left[-\frac{1}{x}\right]\Big|_1^{12}$

$\boxed{\left(-\frac{1}{3}\ln 12\right) + 4\left[-\frac{1}{12} + 1\right]}$